Int. J. Heat Mass Transfer. Vol. 28, No. 8, pp. 1517–1523, 1985
Printed in Great Britain

0017–9310/85 $3.00 + 0.00

Heat transfer of a laminar flow passing a wedge at small Prandtl number: a new approach

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(*Received* 18 *September* 1984 *and in final form* 13 *December* 1984)

Abstract—A new approach based on a two-region model is proposed to analyze the heat transfer problem of a laminar flow passing a wedge at small Prandtl number. The asymptotic local Nusselt number is obtained as

$$Nu_x = \left(\frac{2}{2-\beta}\right)^{1/2}\left(\frac{Re_x\,Pr}{\pi}\right)^{1/2}\Bigg/\left[1+a\left(\frac{2}{2-\beta}\right)^{1/2}\left(\frac{Pr}{\pi}\right)^{1/2}\right]$$

where a is a constant determined by the displacement thickness, and β is the angle factor of the wedge. For $0 \leqslant Pr \leqslant 0.1$, predictions by this expression are within 3.2% for all values of β, while the maximum error is 54% by the conventional asymptote.

INTRODUCTION

LAMINAR flow passing a wedge was analyzed in the early 1930s by Falkner and Skan [1]. With similarity transformation the boundary-layer equation is reduced to an ordinary differential equation, which is well-known as the Falkner–Skan equation. The solution of this equation was analyzed later by Hartree [2] and Stewartson [3].

When the temperature distribution on the wall also satisfies a power law variation, the energy equation can be deduced in a similar way. Solutions for a wide range of Prandtl numbers with constant or variable wall temperature were studied by a number of authors [4–9]. In addition, the asymptotes for very small and very large Prandtl numbers were also studied by some others [10, 11].

Although the exact solution of this heat transfer problem has been found numerically [7, 8], some efforts were also made to find explicit expressions for moderate Prandtl numbers. Lighthill's solution was successfully extended by perturbation expansions [12, 13] which are valid for $Pr > 1$. However, no further improvement has been found for the asymptote for small Prandtl numbers. This solution is simple but yields sizeable errors for moderately small Prandtl numbers. It is found that these errors arise mainly from a physical inconsistency.

The objective of this work is to improve the conventional asymptote for small Prandtl numbers by a simple two-region model to eliminate the physical inconsistency. An explicit equation for the local Nusselt number is obtained with very good accuracy for Prandtl numbers less than 0.1, for all possible accelerating and decelerating flows. The solution converges rapidly to the exact solution as the Prandtl number decreases, and becomes exact at zero Prandtl number.

FORMULATION OF THE PROBLEM

For constant physical properties, the equation of continuity is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

where x is the direction along the wall of the wedge and y is at right angles to it; u and v are the velocity components in x- and y-directions, respectively. The Prandtl boundary-layer equation is

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_\infty\frac{\mathrm{d}u_\infty}{\mathrm{d}x} + \nu\frac{\partial^2 u}{\partial y^2} \tag{2}$$

where $u_\infty(x)$ is the solution of the potential flow. For a wedge with an angle of $\pi\beta$, the potential flow is

$$u_\infty(x) = cx^m \tag{3}$$

where c is a constant and the exponent m is related to the angle factor β by

$$m = \frac{\beta}{2-\beta}. \tag{4}$$

If heat generation due to viscous dissipation is negligible (Appendix A), the energy equation is given by

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}. \tag{5}$$

The boundary conditions are

$$\begin{aligned} y &= 0, \quad u = 0,\ v = 0, \quad T = T_w \\ y &\to \infty, \quad u = u_\infty(x), \quad T = T_\infty. \end{aligned} \tag{6}$$

THE CONVENTIONAL ASYMPTOTE FOR SMALL PRANDTL NUMBERS

Since the thermal boundary layer is thicker than the velocity boundary layer for cases of small Prandtl

NOMENCLATURE

a constant defined by equation (17)
c constant defined by equation (3) $[\mathrm{m}^{1-m}\,\mathrm{s}^{-1}]$
f dimensionless stream function
h heat transfer coefficient $[\mathrm{J\,m^{-2}\,s^{-1}\,K^{-1}}]$
k thermal conductivity $[\mathrm{J\,m^{-1}\,s^{-1}\,K^{-1}}]$
m exponent defined by equation (3)
Nu Nusselt number
Pr Prandtl number
q_y heat flux in the y-direction $[\mathrm{J\,m^{-2}\,s^{-1}}]$
Re Reynolds number
T temperature [K]
u velocity component in the x-direction $[\mathrm{m\,s^{-1}}]$
v velocity component in the y-direction $[\mathrm{m\,s^{-1}}]$
x coordinate along the plate [m]
y coordinate normal to the plate [m].

Greek symbols
α thermal diffusivity $[\mathrm{m^2\,s^{-1}}]$
β angle factor of the wedge
η dimensionless variable defined by equation (28)
ν kinetic viscosity $[\mathrm{m^2\,s^{-1}}]$
θ dimensionless temperature defined by equation (10)
θ_1 dimensionless temperature defined by equation (19)
θ_2 dimensionless temperature defined by equation (21)
ξ dimensionless variable defined by equation (8)
δ_s displacement boundary-layer thickness [m]
ζ dimensionless variable defined by equation (20).

Subscripts
w condition on the wall
x local value at position x
∞ condition in the bulk flow
δ condition at the position of δ_s.

numbers, most of the solution domain for the energy equation is beyond the velocity boundary layer. Thus, the energy equation is simplified by replacing the velocity field by the potential flow. With equations (1) and (3), equation (5) becomes

$$u_\infty \frac{\partial T}{\partial x} - y\frac{\mathrm{d}u_\infty}{\mathrm{d}x}\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}. \tag{7}$$

By introducing a combined variable

$$\xi = \frac{y}{2}(m+1)^{1/2}\left(\frac{u_\infty}{\alpha x}\right)^{1/2} \tag{8}$$

equation (7) is further reduced to an ordinary differential equation

$$\frac{\mathrm{d}^2 T}{\mathrm{d}\xi^2} + 2\xi\frac{\mathrm{d}T}{\mathrm{d}\xi} = 0 \tag{9}$$

and the solution with boundary conditions of equation (6) is

$$\theta = \frac{T-T_\infty}{T_w-T_\infty} = 1 - \frac{2}{\sqrt{\pi}}\int_0^\xi \exp(-\xi^2)\,\mathrm{d}\xi. \tag{10}$$

The local heat transfer coefficient is obtained by the temperature gradient at wall,

$$q_y|_{y=0} = -k\left.\frac{\partial T}{\partial y}\right|_{y=0} = h_x(T_w-T_\infty) \tag{11}$$

and the local Nusselt number becomes

$$Nu_x = \frac{h_x x}{k} = (m+1)^{1/2}\left(\frac{Re_x\,Pr}{\pi}\right)^{1/2} = \left(\frac{2}{2-\beta}\right)^{1/2}\left(\frac{Re_x\,Pr}{\pi}\right)^{1/2} \tag{12}$$

where

$$Re_x = \frac{u_\infty(x)x}{\nu}. \tag{13}$$

However, the procedure of deducing equation (12) is physically inconsistent. In the simplified energy equation, equation (7), the velocity field is replaced by the potential flow. This is a good approximation for the region beyond the velocity boundary layer and the solution, equation (10), should apply only in this region. However, in equation (11) the heat flux is estimated by the temperature gradient on the wedge surface, which is definitely inside the velocity boundary layer. This inconsistency persists as long as the velocity boundary layer is finite, i.e. the Prandtl number is non-zero.

THE TWO-REGION MODEL

From the preceding discussion, it has been indicated that heat transfer inside the velocity boundary layer is not governed by equation (7) and should be modelled separately. Thus, the solution domain in the y-direction is divided into two regions, with the velocity boundary layer thickness, δ_s, as the boundary. The first region is from the wall to the edge of the velocity boundary layer and the second region is from the edge to infinite extension. Energy equations for these two regions are deduced separately as in the following.

For the second region, $\delta_s \leqslant y < \infty$, since the whole region is beyond the velocity boundary layer, the potential flow is applied and the energy equation is identical to equation (7),

$$u_\infty\frac{\partial T}{\partial x} - y\frac{\mathrm{d}u_\infty}{\mathrm{d}x}\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}, \quad \delta_s \leqslant y < \infty. \tag{14}$$

For the first region, $0 \leqslant y \leqslant \delta_s$, the energy equation is deduced in a different sense. By order of magnitude analysis (Appendix A) the orders of the two convective terms are both unity, while the conductive term is $1/Pr$. For small Prandtl numbers the convective terms become negligible and the energy equation is simplified to a conduction equation,

$$\alpha \frac{\partial^2 T}{\partial y^2} = 0, \quad 0 \leqslant y \leqslant \delta_s. \tag{15}$$

The boundary-layer thickness, δ_s, is determined by requiring that the integral mass flux of the deduced energy equations, equations (14) and (15), should be consistent with the actual velocity field,

$$\int_0^{\delta_s} 0 \, dy + \int_{\delta_s}^{\infty} u_\infty \, dy = \int_0^{\infty} u \, dy$$

or

$$\delta_s = \int_0^{\infty} \left(1 - \frac{u}{u_\infty}\right) dy \tag{16}$$

which is the displacement thickness. When the exact solution of the velocity profile is applied, equation (16) is integrated to

$$\delta_s = ax \, Re_x^{-1/2}. \tag{17}$$

Values of the constant a for various wedge angles are listed in Table 1.

By introducing a pseudo-boundary condition

$$y = \delta_s, \quad T = T_\delta \tag{18}$$

the temperature profile in the first region is

$$\theta_1 = \frac{T - T_\delta}{T_w - T_\delta} = 1 - \frac{y}{\delta_s}. \tag{19}$$

Table 1. Values of the constant a for different wedge angles

β	m	a
−0.198838	−0.0904287	3.49781
−0.19	−0.0867580	2.96973
−0.18	−0.0825688	2.76335
−0.16	−0.0740741	2.50825
−0.14	−0.0654206	2.33460
−0.10	−0.0476191	2.09211
−0.05	−0.0243902	1.87901
0.0	0.0	1.72079
0.05	0.0256410	1.59434
0.10	0.0526316	1.48912
0.20	0.111111	1.32039
0.30	0.176471	1.18779
0.40	0.25	1.07850
0.50	0.333333	0.985368
0.60	0.428571	0.903944
0.80	0.666667	0.765372
1.0	1	0.647892
1.2	1.5	0.542828
1.6	4	0.344058
2.0	∞	0

By introducing another combined variable

$$\zeta = \frac{y - \delta_s}{2} (m+1)^{1/2} \left(\frac{u_\infty}{\alpha x}\right)^{1/2} = \xi - \frac{a}{2} (m+1)^{1/2} \, Pr^{1/2} \tag{20}$$

the energy equation of the second region, equation (14), is reduced to an ordinary differential equation of exactly the same form as equation (9). The temperature profile in this region becomes

$$\theta_2 = \frac{T - T_\infty}{T_\delta - T_\infty} = 1 - \frac{2}{\sqrt{\pi}} \int_0^{\zeta} \exp(-\zeta^2) \, d\zeta. \tag{21}$$

The pseudo-boundary condition, equation (18) is eliminated by requiring that the heat flux be continuous at $y = \delta_s$. From equation (19),

$$q_y|_{y=\delta_s} = \frac{k}{\delta_s} (T_w - T_\delta). \tag{22}$$

From equation (21),

$$q_y|_{y=\delta_s} = \frac{k}{x} (T_\delta - T_\infty)(m+1)^{1/2} \left(\frac{Re_x \, Pr}{\pi}\right)^{1/2}. \tag{23}$$

Thus,

$$\frac{T_w - T_\infty}{T_\delta - T_\infty} = 1 + \frac{T_w - T_\delta}{T_\delta - T_\infty} = 1 + \frac{\delta_s}{x} (m+1)^{1/2} \left(\frac{Re_x \, Pr}{\pi}\right)^{1/2} = 1 + a(m+1)^{1/2} \left(\frac{Pr}{\pi}\right)^{1/2}. \tag{24}$$

The local heat transfer coefficient is estimated by

$$q_y|_{y=0} = \frac{k}{\delta_s} (T_w - T_\delta) = h_x (T_w - T_\infty) \tag{25}$$

and the local Nusselt number becomes

$$Nu_x = (m+1)^{1/2} \left(\frac{Re_x \, Pr}{\pi}\right)^{1/2} \div \left[1 + a(m+1)^{1/2} \left(\frac{Pr}{\pi}\right)^{1/2}\right]. \tag{26}$$

This expression can also be written in terms of β by replacing $(m+1)$ by $2/(2-\beta)$.

THE EXACT SOLUTION

The exact solution of equation (5) with the boundary conditions of equation (6) is given [9] by

$$\frac{Nu_x}{Re_x^{1/2}} = \left(\frac{m+1}{2}\right)^{1/2} \times \left\{\int_0^{\infty} \exp\left[-Pr \int_0^{\eta} f(\eta) \, d\eta\right] d\eta\right\}^{-1} \tag{27}$$

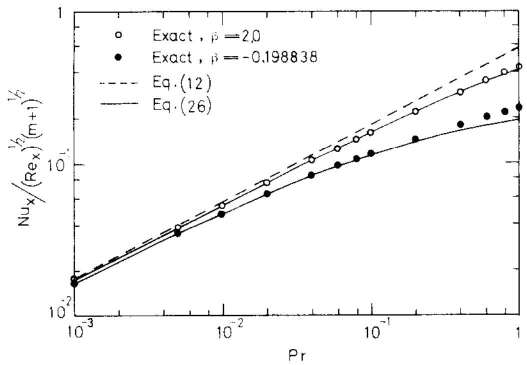

FIG. 1. Comparison of the exact solution and equations (12) and (26) for two extreme values of β.

where

$$\eta = y\left(\frac{u_\infty}{\nu x}\right)^{1/2}\left(\frac{m+1}{2}\right)^{1/2} \qquad (28)$$

and $f(\eta)$ is the dimensionless stream function for the velocity boundary-layer equation, equation (2).

COMPARISON AND DISCUSSION

Solutions of equations (12), (26) and (27) are calculated for the range $0 \leqslant Pr \leqslant 1.0$, with various values of β between -0.198838 and 2. The lower limit of β corresponds to when the velocity boundary layer is about to separate. The upper limit of β corresponds to the instantly accelerating flow, $m = \infty$. Physically, this extreme case does not really exist since, as the wedge increases to 2π, $\beta \to 2$, the space for the fluid shrinks to zero. Nevertheless, only solutions of the two extreme values of β are plotted in Fig. 1 for comparison. Solutions with β values in between are not shown since their results are in between these two cases.

In this figure, the local Nusselt number is adjusted by a parameter $(1+m)^{1/2}$ so that equation (12) yields only one straight broken line. The exact solution is represented by open and dark circles for the upper and lower limiting cases, respectively, and equation (26) by two continuous lines. It should be kept in mind that this figure is plotted in a logarithm scale; therefore, the actual deviation is much larger than it appears.

For the upper limit, $\beta = 2$, Nusselt numbers given by equations (12), (26) and (27) all diverge since they all have the same multiplying factor $(m+1)^{1/2}$. Nevertheless, these three equations still have different dependence on the Prandtl number as they appear in Fig. 1. For equation (26) although the constant a given in Table 1 vanishes in this extreme, the value of $a(m+1)^{1/2}$ remains finite. Thus, equation (26) still differs from equation (12).

In Fig. 1, it is seen that equation (12) always overshoots and the discrepancy increases as the Prandtl number increases. The discrepancy is especially large for the case of $\beta = -0.198838$. On the other hand, equation (26) follows the exact solution quite well for both cases. For $\beta = 2$, equation (26) and the exact solution are almost indistinguishable over the whole range. Even for $\beta = -0.198838$, equation (26) still shows little discrepancy up to $Pr = 0.1$ and small deviation at $Pr = 1.0$.

For a closer comparison, numerical data of the two extreme cases with relative errors are listed in Tables 2 and 3. Calculations for the special case of the flat plate, $\beta = 0$, are also tabulated in Table 4. It becomes quite clear that the two-region model is far better than the conventional asymptote. The relative errors are reduced by about an order with very little calculating effort. It should also be noticed that as the Prandtl number decreases, equation (26) converges much more rapidly to the exact solution, compared to equation (12). These two equations converge at different rates because they approach the exact solution in different senses. For the conventional asymptote the approach senses that the velocity boundary-layer thickness becomes infinitesimal compared to the thermal boundary layer, as the Prandtl number approaches zero. For the two-region model it senses that the simplification of ignoring the convective terms in the first region becomes exact.

Table 2. Comparison of numerical values of $Nu_x/(Re_x)^{1/2}(m+1)^{1/2}$ for $\beta = -0.198838$

Pr	Exact	Equation (26)	Error (%)	Equation (12)	Error (%)
1(−4)	0.553782(−2)	0.553768(−2)	0.003	0.564190(−2)	1.9
5(−4)	0.121091(−1)	0.121062(−1)	0.02	0.126157(−1)	4.2
1(−3)	0.168473(−1)	0.168390(−1)	0.05	0.178412(−1)	5.9
5(−3)	0.352904(−1)	0.352085(−1)	0.2	0.398942(−1)	13
1(−2)	0.476928(−1)	0.474824(−1)	0.4	0.564190(−1)	18
2(−2)	0.635403(−1)	0.630158(−1)	0.8	0.797885(−1)	26
4(−2)	0.832342(−1)	0.819795(−1)	1.5	0.112838	36
6(−2)	0.966346(−1)	0.945903(−3)	2.1	0.138198	43
8(−2)	0.107000	0.104140	2.7	0.159577	49
0.1	0.115531	0.111845	3.2	0.178412	54
0.2	0.144798	0.137000	5.4	0.252313	74
0.4	0.178397	0.162909	8.7	0.356825	100
0.6	0.200240	0.177805	11	0.431079	118
0.8	0.216831	0.188056	13	0.504627	133
1.0	0.231383	0.195757	15	0.564190	145

Table 3. Comparison of numerical values of $Nu_x/(Re_x)^{1/2}(m+1)^{1/2}$ for $\beta = 2$

Pr	Exact	Equation (26)	Error (%)	Equation (12)	Error (%)
1(−4)	0.561964(−2)	0.561960(−2)	0.8(−3)	0.564190(−2)	0.4
5(−4)	0.125053(−1)	0.125047(−1)	0.2(−2)	0.126157(−1)	0.9
1(−3)	0.176216(−1)	0.176201(−1)	0.9(−2)	0.178412(−1)	1.2
5(−3)	0.388229(−1)	0.388052(−1)	0.5(−1)	0.398942(−1)	2.8
1(−2)	0.543146(−1)	0.542653(−1)	0.9(−1)	0.564190(−1)	3.9
2(−2)	0.756823(−1)	0.755481(−1)	0.2	0.797855(−1)	5.4
4(−2)	0.104901	0.104540	0.3	0.112838	7.6
6(−2)	0.126591	0.125953	0.5	0.138198	9.2
8(−2)	0.144422	0.143471	0.7	0.159577	10
0.1	0.159808	0.158517	0.8	0.178412	12
0.2	0.217549	0.214280	1.5	0.252313	16
0.4	0.193201	0.285229	2.7	0.356825	22
0.6	0.347405	0.334259	3.8	0.437019	26
0.8	0.390965	0.372423	4.7	0.504627	29
1.0	0.427941	0.403892	5.6	0.564190	32

Table 4. Comparison of numerical values of $Nu_x/(Re_x)^{1/2}(m+1)^{1/2}$ for $\beta = 0$

Pr	Exact	Equation (26)	Error (%)	Equation (11)	Error (%)
1(−4)	0.558773(−2)	0.558765(−2)	0.0015	0.564190(−2)	0.97
1(−3)	0.173156(−1)	0.173098(−1)	0.033	0.178412(−1)	3.0
5(−3)	0.373922(−1)	0.373314(−1)	0.16	0.398942(−1)	6.7
1(−2)	0.515884(−1)	0.514262(−1)	0.31	0.564190(−1)	9.4
2(−2)	0.705811(−1)	0.701560(−1)	0.60	0.797885(−1)	13
4(−2)	0.955733(−1)	0.944905(−1)	1.1	0.112834	18
6(−2)	0.113489	0.111647	1.6	0.138198	22
8(−2)	0.127860	0.125751	2.1	0.159577	23
0.1	0.140029	0.136504	2.5	0.178412	27
0.2	0.184096	0.175929	4.4	0.252313	37
0.4	0.238927	0.221078	7.5	0.356825	49
0.6	0.276956	0.249237	9.9	0.437019	58
0.8	0.306916	0.270090	12	0.504627	64
1.0	0.332057	0.289617	13	0.564190	70

It is interesting to note that the two-region model always approaches the exact solution from below, while the conventional asymptote always from above. This is because the convective heat transfer terms are totally ignored in the first region for the two-region model and the heat transfer rate is underestimated. On the other hand, for the conventional asymptote the velocity field is replaced by the potential flow and the heat transfer rate is overestimated.

Figure 1 also shows that both equations have better predictions for the accelerating flow. To demonstrate this clearly, predictions at $Pr = 0.1$ with increasing values of β are listed in Table 5. It shows that the relative errors for both equations decrease monotonously as β increases. This is because the velocity boundary layer becomes thinner as the flow converts from decelerating to accelerating. This characteristic of decreasing boundary-layer thickness is shown clearly in Table 1, since the constant a serves as a direct indicator for the velocity boundary-layer thickness.

It should be emphasized that the prediction by the two-region model is not very sensitive to the constant a. Thus, its value need not be calculated by the exact velocity profile. The value by the momentum integral equation gives similarly satisfactory results as well.

CONCLUSION

The two-region model gives satisfactory results for the generalized heat transfer problem of a laminar flow passing a wedge at small Prandtl number. The generalized expression for the local Nusselt number is very accurate for all values of β. In the range of $0 \leqslant Pr \leqslant 0.1$, it is quite safe to replace the exact solution by the simple analytical expression of equation (26) without losing its accuracy. The maximum error is only 3.2% at $Pr = 0.1$ for $\beta = -0.198838$, while errors of other values of β at $Pr = 0.1$ are smaller. Comparing to the maximum error of 54% by the conventional asymptote, the improvement is quite significant. The solution converges very rapidly to the exact solution as the Prandtl number becomes smaller. Predictions for

Table 5. Comparison of numerical values of $Nu_x/(Re_x)^{1/2}(m+1)^{1/2}$ with relative errors at $Pr = 0.1$

β	Exact	Equation (26)	Error (%)	Equation (12)	Error (%)
−0.198838	0.115531	0.111846	3.2	0.178412	54
−0.19	0.122433	0.118442	3.2	0.178412	46
−0.18	0.125224	0.121186	3.2	0.178412	42
−0.16	0.128745	0.124711	3.1	0.178412	39
−0.14	0.131183	0.127195	3.0	0.178412	36
−0.10	0.134660	0.130776	2.9	0.178412	32
−0.05	0.137720	0.134031	2.7	0.178412	30
0.0	0.140029	0.136504	2.5	0.178412	27
0.05	0.141883	0.138511	2.4	0.178412	26
0.10	0.143429		2.3	0.178412	24
0.20	0.145904	0.142922	2.0	0.178412	22
0.30	0.147838	0.145068	1.9	0.178412	21
0.40	0.149414	0.146826	1.7	0.178412	19
0.50	0.150736	0.148306	1.6	0.178412	18
0.60	0.151870	0.149579	1.5	0.178412	17
0.80	0.153731	0.151674	1.3	0.178412	16
1.0	0.155212	0.153345	1.2	0.178412	15
1.2	0.156433	0.154720	1.1	0.178412	14
1.6	0.158349	0.156879	0.9	0.178412	13
2.0	0.159808	0.158517	0.8	0.178412	12

accelerating flows are better since the velocity boundary layers are thinner. For $\beta = 0$, equation (26) reduces to the special case of a flat plate.

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APPENDIX

In the region of $0 \leqslant y \leqslant \delta_s$, by choosing a characteristic velocity u_∞ and a characteristic length L along the plate, the dimensionless equation of continuity is

$$\begin{array}{ccc} \dfrac{\partial u^*}{\partial x^*} & + \dfrac{\partial v^*}{\partial y^*} & = 0 \\ 1/1 & \delta_s^*/\delta_s^* & \end{array} \tag{A1}$$

where * denotes dimensionless variables. The dimensionless terms, 1 or δ_s^*, below the equation denote the order of magnitude of the corresponding terms appear in the equation.

Similarly, when normalized by the inertia term, the dimensionless momentum balance is

$$\begin{array}{cccc} u^*\dfrac{\partial u^*}{\partial x^*} & + v^*\dfrac{\partial u^*}{\partial y^*} & = & \dfrac{1}{Re_L}\left(\dfrac{\partial^2 u^*}{\partial y^{*2}}\right) \\ 1\ \ 1/1 & \delta_s^* 1/\delta_s^* & & 1/(\delta_s^*)^2. \end{array} \tag{A2}$$

Thus, $1/Re_L$ must be order of $(\delta_s^*)^2$.

If viscous dissipation is retained, the energy equation is

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho c_p}\Phi \tag{A3}$$

where Φ is the dissipation function. For the region beyond the velocity boundary layer, $y > \delta_s$, the dissipation term is always negligible since velocity gradient vanishes. Thus, equation (A3) simply reduces to equation (5).

For the region within the velocity boundary layer, the dimensionless energy equation is [14]

$$\begin{array}{ccccc} u^*\dfrac{\partial T^*}{\partial x^*} & + v^*\dfrac{\partial T^*}{\partial y^*} & = & \dfrac{1}{Re_L\,Pr}\dfrac{\partial^2 T^*}{\partial y^{*2}} & + \dfrac{E}{Re_L}\left(\dfrac{\partial u^*}{\partial y^*}\right)^2 \\ 1\ \ 1/1 & \delta_s^*\,1/\delta_s^* & & 1/(\delta_s^*)^2 & (1/\delta_s^*)^2 \end{array} \tag{A4}$$

where $E = u_\infty^2/c_p(T_w - T_\infty)$ = Eckert number.

Since $1/Re_L$ is order of $(\delta_s^*)^2$, the orders of the conduction and dissipation terms are $1/Pr$ and E, respectively. Thus, dissipation term is important only if the Eckert number is order of unity, which usually happens in high velocity systems.

TRANSFERT THERMIQUE D'UN ECOULEMENT LAMINAIRE PASSANT SUR UN DIEDRE, A FAIBLE NOMBRE DE PRANDTL; UNE NOUVELLE APPROCHE

Résumé—On propose une nouvelle approche basée sur un modèle à deux régions pour analyser le problème du transfert thermique d'un écoulement laminaire autour d'un dièdre, à faible nombre de Prandtl. Le nombre de Nusselt asymptotique local est obtenu par :

$$Nu_x = \left(\frac{2}{2-\beta}\right)^{1/2}\left(\frac{Re_x Pr}{\pi}\right)^{1/2} \Big/ \left[1+a\left(\frac{2}{2-\beta}\right)^{1/2}\left(\frac{Pr}{\pi}\right)^{1/2}\right],$$

où a est une constante déterminée par l'épaisseur de déplacement et β l'angle du dièdre. Pour $0 \leqslant Pr \leqslant 0,1$, les prévisions données par cette expression sont à 3,2% près pour toutes les valeurs de β, tandis que l'erreur maximale est 54% par l'asymptote conventionnelle.

WÄRMEÜBERTRAGUNG IN EINER LAMINAREN, UM EINEN KEIL GEFÜHRTEN STRÖMUNG FÜR KLEINE PRANDTL-ZAHLEN—EIN NEUER ANSATZ

Zusammenfassung—Ein auf einem Zwei-Regionen-Modell basierender neuer Ansatz wird vorgeschlagen, um das Problem der Wärmeübertragung einer laminaren, um einen Keil geführten Strömung für kleine Prandtl-Zahlen zu untersuchen. Als örtliche Nusselt-Zahl ergibt sich

$$Nu_x = \left(\frac{2}{2-\beta}\right)^{1/2}\left(\frac{Re_x Pr}{\pi}\right)^{1/2} \Big/ \left[1+a\left(\frac{2}{2-\beta}\right)^{1/2}\left(\frac{Pr}{\pi}\right)^{1/2}\right],$$

wobei a eine aus der Verdrängungsdicke bestimmte Konstante und β der Winkelfaktor des Keils ist. Die Berechnungen mit dieser Gleichung liegen für $0 \leqslant Pe \leqslant 0{,}1$ innerhalb von 3,2% für sämtliche β-Werte, während der maximale Fehler mit dem herkömmlichen Verfahren 54% beträgt.

ТЕПЛООБМЕН В ЛАМИНАРНОМ ПОТОКЕ, ОБТЕКАЮЩЕМ КЛИН ПРИ МАЛОМ ЧИСЛЕ ПРАНДТЛЯ. НОВЫЙ ПОДХОД

Аннотация—Представлен новый, основанный на модели двух зон, метод расчета теплообмена в ламинарном потоке, обтекающем клин при малом числе Прандтля. Для местного числа Нуссельта получено асимптотическое выражение

$$Nu_x = \left(\frac{2}{2-\beta}\right)^{1/2}\left(\frac{Re_x Pr}{\pi}\right)^{1/2} \Big/ \left[1+a\left(\frac{2}{2-\beta}\right)^{1/2}\left(\frac{Pr}{\pi}\right)^{1/2}\right],$$

где a—константа, определяемая толщиной сдвига, β—угол клина. Для $0 \leq Pr \leq 0{,}1$ и любых β ошибка в расчетах по этому выражению не превышала 3,2%, в то время как обычная асимптотика дает максимальную погрешность 54%.